Dynamic Game for Regional Climate Mitigation Control

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Abstract: One of the most widely used models for studying the geographical economics of climate change is the Regional Integrated model of Climate and the Economy (RICE). In this paper, we investigate how cooperation and non-cooperation arise in climate policy across regions under the RICE framework from the standpoints of game theory and optimal control. We show that the RICE model is inherently a dynamic game. We study both cooperative and non-cooperative solutions to this RICE dynamic game. Our results show how game theory may be used to help international negotiations reach an agreement on regional climate-change mitigation strategies, as well as how cooperative and competitive regional relationships impact future climate change.

Keywords: Economics of Climate Change, Game Theory, Optimal Control

1. INTRODUCTION

The issue of global warming has emerged as a central international environmental question over decades. As a consequence of economic development, human-caused emissions of greenhouse gases (GHG), especially carbon dioxide (CO_2), contribute to a significant increase in global temperature by 1.15°C relative to the pre-industrial age (Pörtner et al., 2022). This temperature deviation yields significant changes in the global climate, including larger wildfires, sea level rise and melting of ice lands. To assess the damages of anthropogenic GHG, especially CO_2, scientists employ the Integrated Assessment Models (IAMs) (Hope, 2013; Nordhaus and Sztorc, 2013). IAMs simulate the dynamics of economy-climate interactions by incorporating mathematical models from both economics and geophysical science. The Dynamic Integrated model of Climate and Economy (DICE) is one of the most well-known IAMs (Nordhaus and Sztorc, 2013). The DICE model is a globally aggregated model and treats global warming as a single-agent problem. Considering the crucial aspect of regional socio-economic heterogeneity, the Regional Integrated model of Climate and the Economy (RICE) was proposed, which is a decentralized version of the DICE model (Nordhaus, 2010). By dividing the world into several regions, the RICE model takes the vantage point in determining how multiple regions may jointly design climate policies and cope with global warming issue.

In the RICE model, there are two coupled but conflicting sides in regional emission-reduction policies: regions are affected by the same global climate system; they also decide their climate strategies to benefit their individual economic benefits and political self-interests. Therefore, it is actually a decision-making process where non-cooperation occurs. Game theory has been a fundamental tool in explaining decision-making of self-interested players in a competitive setting (Owen, 2013). In a game, each player takes its action to maximize its payoff function. The payoff function of each player relies on not only its own action, but also other players' actions, resulting in inherent non-cooperation. In the RICE model, regional climate strategies are associated with a common global climate dynamic, and as a result, climate policy decisions fall into the concept of dynamic games (Başar and Olsder, 1998). In a dynamic game, the strategic interaction among players recurs over time. The group of players is associated with a dynamical game state that depends on all players' actions. The goal is for each player to take an action to maximize each player's cumulative payoff function over time that depends on the game state, its own action, and other players' actions.

In this paper, we investigate how cooperation and non-cooperation in regional climate policies under the RICE framework affect the formation of international climate treaties, the implementation of regional climate-change mitigation measures, and the resulting implications of competitive decisions for climate change from the perspective of game theory and optimal control. In control community, there are a few efforts on studying the climate-change mitigation measures. The work of (Kellett et al., 2019) provided a tutorial introduction to the DICE model and proposed a receding horizon approach to DICE. A bi-objective optimal control problem (OCP) on DICE was studied, objectives of which are maximizing social welfare and minimizing atmospheric temperature deviation (Heris and Rahnamayan, 2020). The work of (Carlino et al., 2020) studied a multi-objective stochastic OCP on DICE, which accounts for stochastic disturbances and aligns with physical targets posed by international agreements on climate change mitigation.

The contributions of this work are summarized as follows. We show that the RICE model is inherently a dynamic game, termed the RICE game. Both cooperative and non-cooperative solutions to the RICE game are considered:

- For cooperative solutions, we study global social welfare maximization problem.Next, we classify regions into two clusters of developed regions and developing regions, and examine the social welfare Pareto frontier under the concept of Pareto optimality.
- For non-cooperative solutions, we study best-response dynamics and open-loop Nash equilibrium of the RICE game. A Recursive Best-response Algorithm for Dynamic Games (RBA-DG) is proposed. By applying it to the RICE game, the simulation result shows that the obtained sequence of actions converges to a steady point, indicating that RBA-DG is useful for computing the open-loop Nash equilibrium of the RICE game.

Our implementations of various solution concepts for RICE under the proposed dynamic-game perspective are developed relying on previous efforts (Faulwasser et al., 2018; Kellett et al., 2016; Anthoff and Errickson, 2021). We have open-sourced our implementation as a RICE-GAME framework, with a Matlab and Casadi-based implementation of RICE dynamic game, Preprint at `arXiv.org`, code for download at (Chen and Shi, 2022).

The paper is organized as follows. Section 2 provides preliminaries of the DICE/RICE model and dynamic games. Section 3 shows the RICE model as a dynamic game. Section 4 considers cooperative solutions for RICE game. Best-response dynamics and open-loop Nash equilibrium for RICE game are studied in Section 5. The paper ends with concluding remarks in Section 6.

2. PRELIMINARIES

In this section, we introduce some preliminary knowledge on the DICE/RICE model and dynamic games.

2.1 The DICE Model

The DICE model (Nordhaus and Sztorc, 2013) simulates the interplay between economy and climate. The DICE model is composed of two sectors: a geophysical sector that accounts for global interaction between carbon and temperature, and an economic sector that is globally aggregated for the world total.

Geophysical and Economic Sectors. In the geophysical sector, the DICE model considers CO_2 emissions as the major contributor to climate change. The geophysical sector is constructed as follows.

- There are two main sources of CO_2 emissions: industrial CO_2 emissions related to the carbon intensity (denoted by σ) of global economic activities and natural CO_2 emissions due to land use changes, E^{land}. Global CO_2 emissions as the sum of industrial and natural emissions drive the carbon cycle of the Earth.
- The carbon dynamics are described by a three-reservoir model (Post et al., 1990) on the carbon flows among three reservoirs: the atmosphere, the upper oceans and biosphere, and the deep oceans. The average carbon masses in those reservoirs are represented by M^{AT}, M^{UP}, and M^{LO}, respectively.
- Accumulations of CO_2 emissions and other GHG warm the Earth's surface through enhanced radiative forcing. Radiative forcing resulted by CO_2 emissions has a dependence on the atmospheric carbon mass; GHG other than CO_2 emissions contribute to exogenous radiative forcing F^{EX}.
- The rise in atmospheric temperature is driven by radiative forcing. Temperature dynamics are captured by a two-layer model (Schneider and Thompson, 1981). Given the temperature in year 1750 as zero reference, T^{AT} and T^{LO} represent the temperature deviation in the atmosphere and in the lower ocean from those of the reference year, respectively.

The economic sector of DICE is based on the Cobb-Douglas production function (Cobb and Douglas, 1928), where gross economic output is determined by total factor productivity A, labor L, and capital K. Total factor productivity and labor evolve exogenously; capital dynamics follows the Solow-Swan model (Swan, 1956) where capital depreciates over time and is replenished by investment.

Control Inputs. The DICE model assumes two control decisions: the saving rate s and the emission-reduction rate μ. The saving rate s represents the ratio of investment to economic output; the emission-reduction rate μ represents the rate at which industrial CO_2 emissions are reduced.

2.2 The RICE Model

The RICE model is a variant of the DICE model that accounts for regional climate damages and control decisions (Nordhaus, 2011). The RICE-2011 model uses a time-step of 10 years, starting from the year 2005 as the initial year.

The RICE-2011 model integrates a global geophysical sector with regional economic sectors:

- The global geophysical sector of the RICE-2011 model contains the same carbon dynamics and temperature dynamics as the DICE model;
- The regional economic sectors of the RICE-2011 model disaggregates the world into 12 regions, each of which is equipped with region-specific climate damage level, economic factors, and saving rate and emission-reduction rate as local control inputs.

2.3 Dynamic Games

An n-player discrete-time dynamic game over a finite horizon is defined as follows.

Dynamic Game. The n players are indexed in $\mathcal{V} := \{1, 2, \ldots, n\}$; time is discrete with the steps indexed in $\mathcal{T} := \{0, 1, \ldots, T\}$. Each player can manipulate the game through its control decisions, and the control decision space of player $i \in \mathcal{V}$ is denoted by $\mathcal{U}_i \subseteq \mathbb{R}^d$. At each time step $t = 0, \ldots, T$, the decision executed by player i is denoted by $\mathbf{u}_i(t) \in \mathcal{U}_i$. We also use $\mathbf{u}(t) = [\mathbf{u}_1^\top(t); \ldots; \mathbf{u}_n^\top(t)]$, $\mathbf{U}_i = [\mathbf{u}_i^\top(0); \ldots; \mathbf{u}_i^\top(T)]$ and $\mathbf{U} = [\mathbf{U}_1; \ldots; \mathbf{U}_n]$ to represent the all-player decision profile at time t, the player-i decision throughout the time horizon, and the decision profile for all players and for all time steps. The control decisions of all players excluding player i at time step t is denoted by $\mathbf{u}_{-i}(t)$, and the control decisions of all players excluding player i over the entire horizon is represented by $\mathbf{U}_{-i}$.

For each $t \in \mathcal{T}$, the group of players are associated with a dynamical state $\mathbf{x}(t) \subseteq \mathbb{R}^m$ that evolves according to $\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \mathbf{x}(0) = \mathbf{x}_0, t \in \mathcal{T}$, with $\mathbf{x}_0$ being initial state. At each time $t \in \mathcal{T}$, upon $\mathbf{u}_i(t)$, agent i receives payoffs $g_i(\mathbf{x}(t), \mathbf{u}_i(t), \mathbf{u}_{-i}(t)) \in \mathbb{R}$ given other players' actions $\mathbf{u}_{-i}(t)$ and current state $\mathbf{x}(t)$, where $g_i(\mathbf{x}(t), \mathbf{u}_i(t), \mathbf{u}_{-i}(t))$ is a continuous function with respect to $\mathbf{x}(t)$, $\mathbf{u}_i(t)$, and $\mathbf{u}_{-i}(t)$. The cumulative payoff of agent i throughout the time horizon is therefore $\mathsf{J}_i(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i}) = \sum_{t=0}^{T} g_i(\mathbf{x}(t), \mathbf{u}_i(t), \mathbf{u}_{-i}(t))$ where $\mathbf{X} = (\mathbf{x}^\top(0), \ldots, \mathbf{x}^\top(T))^\top$. Each player's goal is to make decisions for maximizing its cumulative payoffs; the system dynamics produces a terminal state $\mathbf{x}(T+1)$ towards the end of the time horizon as a result of those decisions.

Open-loop Nash equilibrium. In the open loop information structure, each player knows the initial state $\mathbf{x}_0$, and then plans at $t = 0$ all the control decisions $\mathbf{u}_i(t)$ for $t \in \mathcal{T}$. Consequently, the open loop control decision of $\mathbf{u}_i(t)$ can be written as $\mathbf{u}_i(t) = \mathbf{u}_i(t, \mathbf{x}_0)$. Denote an open loop decision profile by $\mathbf{U}^*$ where $\mathbf{U}_i^* := [\mathbf{u}_i^*(0, \mathbf{x}_0); \ldots; \mathbf{u}_i^*(T, \mathbf{x}_0)], i \in \mathcal{V}$. We introduce the following definition. Here with slight abuse of notation we also write $\mathsf{J}_i(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i})$ as $\mathsf{J}_i(\mathbf{x}_0, \mathbf{U}_i, \mathbf{U}_{-i})$ noting the fact that $\mathbf{X}$ is uniquely determined by $\mathbf{x}_0$ and $(\mathbf{U}_i, \mathbf{U}_{-i})$.

Definition 1. (Open Loop NE). Given the initial state $\mathbf{x}_0$, a control decision profile $\mathbf{U}^*$ is said to be an open loop Nash equilibrium control decision profile if there holds for all $i \in \mathcal{V}$ and all $\mathbf{U}_i$ that $\mathsf{J}_i(\mathbf{x}_0, \mathbf{U}_i^*, \mathbf{U}_{-i}^*) \geq \mathsf{J}_i(\mathbf{x}_0, \mathbf{U}_i, \mathbf{U}_{-i}^*)$.

3. RICE AS A DYNAMIC GAME

In this section, we show that the RICE model is inherently a dynamic game where regional saving rates and emission-reduction rates regulate global temperature, and then the global temperature has impact on regional social welfares through climate damage. Our presentation is based on the RICE-2011 model with slight modifications, but the nature of being a dynamic game is embedded in all RICE models.

There are 12 regions in the RICE-2011 model. Each region is considered a player and the regions are indexed in $\mathcal{V} = \{1, 2, \ldots, n\}$ with $n = 12$. We operate the RICE dynamic game in periods of 5 years, starting from the year 2020 as the initial year. Taking the discrete time step index $\mathcal{T}$, the relation between an actual calendar year and the corresponding discrete time step is determined by $year(t) = year(0) + 5t, \quad year(0) = 2020$. Note that although most variables in the RICE-2011 model are defined as flows per year and only some variables are in flows per decade (Nordhaus, 2011, Supplementary Material), all variables in this present are defined as flows per year.

3.1 System Dynamics

We define the dynamical state of the RICE model at time step $t \in \mathcal{T}$ as $\mathbf{x}(t) = [T^{\mathrm{AT}}(t); T^{\mathrm{LO}}(t); M^{\mathrm{AT}}(t); M^{\mathrm{UP}}(t); M^{\mathrm{LO}}(t); K_1(t); \ldots; K_n(t)] \in \mathbb{R}^{n+5}$. Let region i's control decision at time step t be $\mathbf{u}_i(t) = [s_i(t); \mu_i(t)]^\top := [\mathbf{u}_{i[1]}(t); \mathbf{u}_{i[2]}(t)]^\top \in [0, 1]^2$. Consequently, the control decisions of the RICE dynamic game at time step $t \in \mathcal{T}$ of all players are $\mathbf{u}(t) = [s_1(t); \mu_1(t); \ldots; s_n(t); \mu_n(t)] \in [0, 1]^{24}$. According to the RICE model, the dynamics of $\mathbf{x}(t)$ can be written as $\mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \quad t \in \mathcal{T}$, where $\mathbf{f} := [f_1; f_2; \ldots; f_{n+5}]^\top$ follows from the interdependency among the geophysical signals and the economic signals, and the feedback between the two sectors. In what follows, we briefly describe the form of entries in the dynamics $\mathbf{f}$. For a more detailed description, please refer to (Nordhaus, 2011).

Carbon dynamics. There are three carbon reservoirs: the atmosphere, the upper oceans and the biosphere, and the deep oceans. The atmospheric carbon reservoir has an additional input, the global CO_2 emissions $E(t)$ that is related to economic activities and land use at time t. The carbon dynamics for carbon transition among the three reservoirs are described by

$$\mathbf{M}(t+1) = \boldsymbol{\zeta}\mathbf{M}(t) + \boldsymbol{\xi}_1 E(t) \tag{1}$$

where $\mathbf{M}(t) = [M^{\mathrm{AT}}(t); M^{\mathrm{UP}}(t); M^{\mathrm{LO}}(t)]$ and $\boldsymbol{\zeta} \in \mathbb{R}^{3\times3}, \boldsymbol{\xi}_1 \in \mathbb{R}^{3\times1}$ represent diffusion parameters.

Temperature dynamics. The evolution of the atmospheric and ocean temperature is governed by

$$\mathbf{T}(t+1) = \boldsymbol{\phi}\mathbf{T}(t) + \boldsymbol{\xi}_2 F(t) \tag{2}$$

where $\mathbf{T}(t) = [T^{\mathrm{AT}}(t); T^{\mathrm{LO}}(t)]$ and $\boldsymbol{\phi} \in \mathbb{R}^{2\times2}, \boldsymbol{\xi}_2 \in \mathbb{R}^{2\times1}$ represent parameters. The radiative forcing at time step t is computed as $F(t) = \eta \log_2(\frac{M^{\mathrm{AT}}(t)}{M^{\mathrm{AT},1750}}) + F^{\mathrm{EX}}(t)$. Here η and $M^{\mathrm{AT},1750}$ are constants, and $F^{\mathrm{EX}}(t)$ represents radiative forcing of other GHG at time step t.

Economic dynamics. The economy of each region $i \in \mathcal{V}$ at time step $t \in \mathcal{T}$ follows from the Cobb-Douglas production function (Cobb and Douglas, 1928), $Y_i(t) = A_i(t)K_i(t)^{\gamma_i}(t)L_i(t)^{1-\gamma_i}$, where $Y_i(t), A_i(t), K_i(t)$ and $L_i(t)$ represent region i's gross economic output, total factor productivity, capital stock and labor at time step t, respectively. Here, $\gamma_i, i \in \mathcal{V}$, are parameters.

Economy-climate feedback. Global CO_2 emissions at time step t are the sum of natural emissions because of each region's land use at time step t, $E_i^{\mathrm{land}}(t)$, and industrial emissions resulted from each region's economic activities at time step t. Each region i's industrial emissions depend on each region i's carbon intensity at time step t, $\sigma_i(t)$, which is an exogenous variable. Consequently, global CO_2 emissions at time step t is described by $E(t) = \sum_{i=1}^{n} \Big(\sigma_i(t)(1-\mu_i(t))Y_i(t) + E_i^{\mathrm{land}}(t)\Big)$, where $\mu_i(t), i \in \mathcal{V}, t \in \mathcal{T}$, are control decisions representing the emission-reduction rate.

The emission abatement cost fraction as the percentage of gross economic output spent on emission-reduction effort at time step t is given by $\Lambda_i(t) = 1 - \theta_i^{[1]}(t)\mu_i(t)^{\theta_i^{[2]}}$, where $\theta_i^{[2]}$, and $\theta_i^{[1]}(t), i \in \mathcal{V}, t \in \mathcal{T}$, are parameters. The parameters $\theta_i^{[1]}(t), i \in \mathcal{V}, t \in \mathcal{T}$, are calculated by $\theta_i^{[1]}(t) = \frac{pb_i}{1000\cdot\theta_i^{[2]}}(1 - \delta_i^{pb})^{t-1}\sigma_i(t)$, where pb_i, represents the price of backstop technology at time step $t = 0$ for region i to replace all carbon fuels, and the $\delta_i^{pb}, i \in \mathcal{V}$, are parameters. The damage function $\Omega_i(t)$ is the percentage of gross economic output damaged by temperature rising. As a result, the net economic output $Q_i(t)$ (economic output after the emission-reduction spending and climate damage), is given by $Q_i(t) = \Omega_i(t)\Lambda_i(t)Y_i(t)$. The Solow-Swan model (Swan, 1956) gives a description of capital accumulation of each region $i \in \mathcal{V}$:

$$K_i(t+1) = (1-\delta_i^{\mathrm{K}})^5 K_i(t) + 5s_i(t)Q_i(t), \tag{3}$$

where $\delta_i^{\mathrm{K}}, i \in \mathcal{V}$, are parameters, and $s_i(t), i \in \mathcal{V}, t \in \mathcal{T}$, are control decisions representing the saving rate.

3.2 Damage Functions

A simplified form of damage function is employed, which only depends on rising atmospheric temperature deviation, $\Omega_i(t) = 1 - a_i^{[1]}T^{\mathrm{AT}}(t) - a_i^{[2]}T^{\mathrm{AT}}(t)^{a_i^{[3]}}$, where $a_i^{[1]}, a_i^{[2]}$, and $a_i^{[3]}, i \in \mathcal{V}$, are parameters calibrated to yield a certain amount of damage loss.

3.3 Payoff Functions

Note that from the net economic output $Q_i(t)$, a total amount of $s_i(t)Q_i(t)$ has been made as investment. The remaining part $C_i(t) = (1-s_i(t))Q_i(t)$ can then be used for consumption. In RICE models, for region i, the social welfare of the population $L_i(t)$ consuming $C_i(t)$ of economic output at time t is defined by the population-weighted utility of per capita consumption $g_i(C_i(t), L_i(t)) = L_i(t) \cdot \frac{(\frac{C_i(t)}{L_i(t)})^{1-\alpha_i}-1}{1-\alpha_i}$, where α_i is a constant. The cumulative social welfare of region i across the time horizon is then given by

$$\mathsf{J}_i = \sum_{t=0}^{T} \frac{g_i(C_i(t), L_i(t))}{(1+\rho_i)^{5t}} := \mathsf{J}_i(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i}) \tag{4}$$

where ρ_i is a constant discounting factor. For each region $i \in \mathcal{V}$, naturally it will attempt to maximize its cumulative social welfare.

We have now formally represented the RICE-2011 model as a dynamic game, termed the *RICE game*. In what follows when we implement the proposed RICE game, the values of the initial state $\mathbf{x}(0)$ and the parameters are updated in the following way: the initial state are calibrated to match the data in year 2020; the parameters in the geophysical sector use the latest updated values in the DICE-2016 model, while the parameters in the regional economic sector remain unchanged.

4. RICE GAME: COOPERATIVE SOLUTIONS

In this section, we study the solutions to the RICE game under cooperative settings.

4.1 RICE Social Welfare Maximization

The RICE-2011 model focused on the sum of the weighed regional social welfare across all regions: $\mathsf{W}_{\mathbf{c}} = \sum_{i=1}^{n} c_i \mathsf{J}_i$, where $0 < c_i < 1$ is know as the Negishi weight for region i with $\sum_{i=1}^{n} = 1$. The values of the c_i were calibrated in the work of (Nordhaus, 2013).

Solution Concept. One benchmark cooperative solution to the RICE game is for a centralized climate policy planner to compute the $\mathbf{U}_1, \ldots, \mathbf{U}_n$ for all regions that achieve the maximal value of $\mathsf{W}_{\mathbf{c}}$, for a given initial condition $\mathbf{x}_0$.

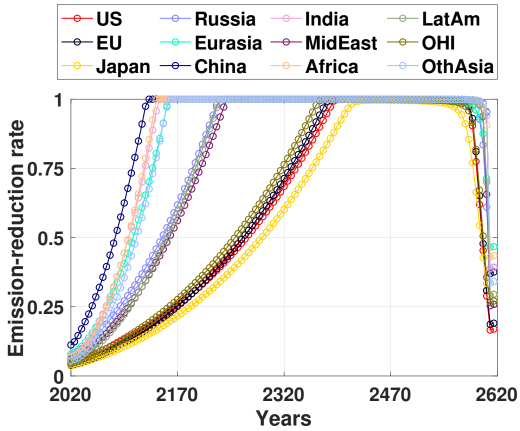

Fig. 1. Optimal emission-reduction rate under $\mathbf{U}^{\mathrm{w}}$.

Definition 2. A decision profile $\mathbf{U}^{\mathrm{w}}$ is a global social welfare equilibrium if it is a solution to the following optimization problem

$$\begin{aligned} \max_{\mathbf{U}_1,\cdots,\mathbf{U}_n} \quad & \mathsf{W}_{\mathbf{c}}(\mathbf{X}, \mathbf{U}) \\ \text{subject to} \quad & \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ & \mathbf{u}(t) \in [0,1]^{24}, \quad t \in \mathcal{T}. \end{aligned} \tag{5}$$

Results. We run a time length of 600 years, and therefore we set $T = 120$. We obtain the global social welfare equilibrium $\mathbf{U}^{\mathrm{w}}$ for the RICE game under Definition 2 by solving the corresponding OCP (5).

First, each region's optimal control decisions in emission-reduction rate under $\mathbf{U}^{\mathrm{w}}$ are plotted in Fig 1. Clearly, regions including China, Africa, India, Eurasia, OthAsia, and Russia have relatively high emission-reduction rates who should reach carbon neutral status relatively sooner. The reason for that could be twofolds. First, regions such as Africa and India bear the greatest damage loss caused by rising atmospheric temperature. Second, it takes regions such as China, Eurasia and Russia comparatively lower cost to reduce carbon emissions.

Then, in Fig. 2, we examine the trajectory of the atmospheric temperature deviation when the emission-reduction rates and saving rates under $\mathbf{U}^{\mathrm{w}}$ are taken. The work of (Nordhaus, 2011) also solved the OCP (5) with the same horizon length of 600 years starting from the year of 2005. Although we are inaccessible to the exact and complete data of results in (Nordhaus, 2011, Fig. 3), the atmospheric temperature deviation trajectory in Fig. 2 appears to be similar with that in (Nordhaus, 2011). To be specific, the atmospheric temperature deviation in (Nordhaus, 2011, Fig. 3) is around $2.8°C$, whereas ours is approximately $3°C$.

4.2 RICE Pareto Frontier

In recent years' international climate policy forums, the divide between developed and developing regions has been one of the main barriers from a global consensus on carbon emission rates (Dimitrov, 2010; Castro, 2021). In this subsection, we focus on the social welfare Pareto frontier between developed and developing regions involved in the RICE game.

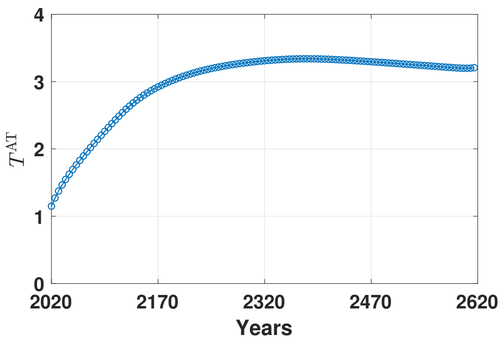

Fig. 2. Atmospheric temperature deviation trajectory under $\mathbf{U}^{\mathrm{w}}$.

Pareto Frontier. We classify the regions in the RICE-2011 model into two clusters of regions: developed regions in $\mathcal{V}_{\text{developed}}$ and developing regions in $\mathcal{V}_{\text{developing}}$. Correspondingly, social welfare of two clusters of regions is defined as $\mathsf{W}_{\text{developed}} = \sum_{i \in \mathcal{V}_{\text{developed}}} \mathsf{J}_i$ and $\mathsf{W}_{\text{developing}} = \sum_{i \in \mathcal{V}_{\text{developing}}} \mathsf{J}_i$, respectively. For the considered RICE game over these two clusters of regions, we consider the Pareto optimality.

Definition 3. For the RICE game under developed and developing clusters of regions, a decision profile $\mathbf{U}^{\mathrm{p}}$ is a Pareto social welfare equilibrium between the developed and developing clusters if there does not exist another decision profile $\mathbf{U}$ such that

(i) there hold
$$\mathsf{W}_{\text{developed}}(\mathbf{X}^{\mathrm{p}}, \mathbf{U}_i^{\mathrm{p}}, \mathbf{U}_{-i}^{\mathrm{p}}) \leq \mathsf{W}_{\text{developed}}(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i}),$$
$$\mathsf{W}_{\text{developing}}(\mathbf{X}^{\mathrm{p}}, \mathbf{U}_i^{\mathrm{p}}, \mathbf{U}_{-i}^{\mathrm{p}}) \leq \mathsf{W}_{\text{developing}}(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i});$$
(ii) either one of the above two inequalities holds strictly.

Here $\mathbf{X}$ and $\mathbf{X}^{\mathrm{p}}$ are the states evolved under $\mathbf{U}$ and $\mathbf{U}^{p}$, respectively.

Based on (Engwerda, 2005, Lemma 6.1), we can calculate such Pareto social welfare frontier between the developed and developing clusters by solving the family of optimization problems for a given initial condition $\mathbf{x}_0$:

$$\begin{aligned} \max_{\mathbf{U}_1, \cdots, \mathbf{U}_n} \quad & p \cdot \mathsf{W}_{developed} + (1-p) \cdot \mathsf{W}_{developing} \\ \text{subject to} \quad & \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \quad \mathbf{x}(0) = \mathbf{x}_0, \\ & \mathbf{u}(t) \in [0,1]^{24}, \quad t \in \mathcal{T}, \end{aligned} \tag{6}$$

where p is selected in the interval $[0,1]$. For any fixed $p \in [0,1]$, we obtain a Pareto solution, and their collection forms the Pareto frontier between the developed and developing clusters. The Pareto formulation might have the potential to serve as a benchmark for the interchange of positions between developed regions and developing regions on climate policies. The parameter p serves as a quantitative characterization to the allocation of responsibility for climate change mitigation between developed regions and developing regions: If p is close to 0, developed regions will take higher responsibility, as the developed regions will if p is close to 1.

Results. We set a time length of 600 years. We take 999 linearly spaced values between 0.001 and 0.999 as the values of p. For each p, we obtain the Pareto solution $\mathbf{U}^{p}$ by solving the respective OCP (6). We plot the social welfare Pareto frontier between developed and developing regions in Fig. 3. We also plot the atmospheric temperature deviation at the final time step, $T^{\mathrm{AT}}(120)$, versus the parameter p in Fig. 4.

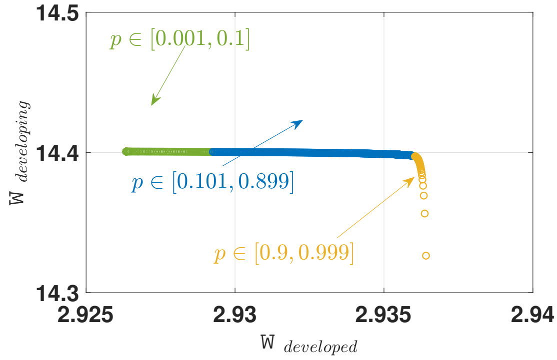

Fig. 3. Social welfare Pareto frontier between developed regions and developing regions.

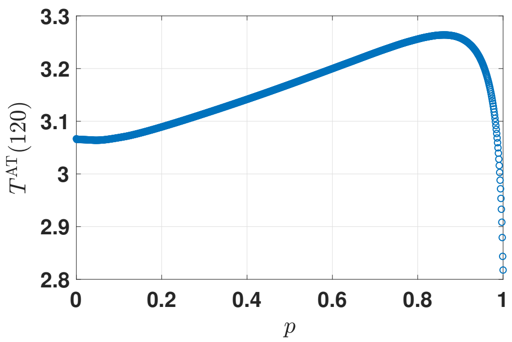

Fig. 4. Atmospheric temperature deviation in the year 2620, $T^{\mathrm{AT}}(120)$, versus the parameter p.

These results reveal a few notable effects.

- From Fig. 3, it can be seen that the values of p can not drastically change the social welfare received for both developing and developed clusters of regions. The maximal ($p = 0.999$) and minimal ($p = 0.001$) values for $\mathsf{W}_{\text{developed}}$ differ only by 0.42%; the maximal ($p = 0.001$) and minimal ($p = 0.999$) values for $\mathsf{W}_{\text{developing}}$ differ only by 0.56%.
- Fig 4 shows that the atmospheric temperature deviation at the final time step is quite robust with respect to p under the Pareto equilibrium. In fact, for all values of p between 0.001 and 0.999, $T^{\mathrm{AT}}(120)$, the atmospheric temperature deviation in the year of 2620, always falls between $2.8°C$ and $3.3°C$.

5. RICE GAME: BEST-RESPONSE DYNAMICS AND OPEN-LOOP NE

In this section, we study the best-response dynamics and open-loop Nash equilibrium of the RICE game.

5.1 Best-response Recursions for Dynamic Games

We now establish the best-response recursions for the dynamic game introduced in Section 2.3 with n players over a finite horizon T. We assume the dynamic game is repeatedly and recursively played for N episodes, where each episode consists of T time steps. We thereby define the aggregated control decisions of all players in episode $k = 1, \cdots, N$ by $\mathbf{U}^{(k)}$, and the decisions of player i in episode k by $\mathbf{U}_i^{(k)}$. Similarly, the decisions of players excluding player i in episode k is denoted by $\mathbf{U}_{-i}^{(k)}$. In view of the best-response for static games (Roughgarden, 2010), the best-response recursion of the agents in the dynamic game over the N episodes are described in the following Recursive Best-response Algorithm for Dynamic Games (RBA-DG) as in Algorithm 1. The RBA-DG produces an output $\mathbf{U}_*^{\mathrm{NE}} = \mathbf{U}^{(N)}$ after N episodes of updates.

Algorithm 1 Recursive Best-response Algorithm for Dynamic Games (RBA-DG)

Input: Episodes N; $c_i, i \in \mathcal{V}$.

compute an optimal cooperative solution $\mathbf{U}^{\mathrm{c}}$ from (5)
let $\mathbf{U}_i^{(0)} = \mathbf{U}_i^{\mathrm{c}}, \forall i \in \mathcal{V}$
while `k < N` **do**
  for `each player` $i \in \mathcal{V}$ **do**
    observe $\mathbf{U}_{-i}^{(k)}$
    compute $\mathbf{U}_i^{(k+1)}$ by solving the problem

$$\max_{\mathbf{U}_i} \quad \mathsf{J}_i(\mathbf{X}, \mathbf{U}_i, \mathbf{U}_{-i}), \tag{7a}$$

$$s.t. \quad \mathbf{x}(t+1) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)), \; \mathbf{x}(0) = \mathbf{x}_0, \tag{7b}$$

$$\mathbf{U}_{-i} = \mathbf{U}_{-i}^{(k)}. \tag{7c}$$

  end for
  $\mathbf{U}^{(k+1)} = [\mathbf{U}_1^{(k+1)}; \cdots; \mathbf{U}_n^{(k+1)}]$
end while
return $\mathbf{U}_*^{\mathrm{NE}} = \mathbf{U}^{(N)}$

The implementation of RBA-DG requires two conditions. First, the initial value of the system $\mathbf{x}(0) = \mathbf{x}_0$ needs to be known by all players at the beginning of the process. Second, at the end of each episode $k = 1, \cdots, N$, every player should be able to observe or know all other players' decision sequences over the episode $\mathbf{U}_{-i}^{(k)}$. Then the update of the player decisions for the next episode follows directly from the best-response dynamics. We present the following result, which holds true immediately from definition of open-loop Nash equilibrium.

Proposition 1. Consider a repeatedly played n-player dynamic game. Let $N = \infty$ in the RBA-DG. Suppose there exists $\mathbf{U}^*$ such that there holds $\lim_{k\to\infty} \mathbf{U}^{(k)} = \mathbf{U}^*$. Then $\mathbf{U}^*$ is an open-loop Nash equilibrium for dynamic game.

5.2 Recursive Best-response for RICE

We propose to adopt RBA-DG over RICE as a mechanism for regional climate policy negotiations. The overall negotiations take a prescribed N episodes. In each round of the negotiations, regions accept RICE as the standing model for climate-economy integration, and decide their emission reduction rates and saving rates for a fixed time horizon. At the end of each round, all regions reveal their current planning of the emission reduction rates and saving rates for the entire time horizon to other regions. Then, during the next round of negotiations, regions get to revise their planned decisions and adopt RBA-DG as their principle of updating such planned decisions. After the N episodes of negotiations, if all regions realize none of them can unilaterally change their climate actions and gain significant increase in social welfare, such a mechanism will produce an approximate open-loop Nash equilibrium for the RICE game in view of Proposition 1. Such a Nash equilibrium holds higher promise of being accepted by all regions since no region is able to benefit from a revised decision when all other regions take the actions from the equilibrium.

Results. Now we implement the RBA-DG over the RICE game. We set the number of episodes to be $N = 21$.

Convergence. In Fig. 6, we plot the trajectory of $\|\mathbf{U}^{(k+1)} - \mathbf{U}^{(k)}\|$ versus episode k. The result shows that when applying RBA-DG over the RICE game, the obtained sequence of $\mathbf{U}^{(k)}$ converges to a steady point, and after 5 episodes, the $\|\mathbf{U}^{(k+1)} - \mathbf{U}^{(k)}\|$ has become very close to zero. From Proposition 1, this implies that the RBA-DG may also serve as an efficient algorithm for computing the open-loop Nash equilibrium of the RICE game.

Cooperation vs Non-cooperation. In Fig. 5, we plot the comparison of each region's optimal emission-reduction rates under $\mathbf{U}^{\mathrm{w}}$ and $\mathbf{U}_*^{\mathrm{NE}}$ obtained by RBA-DG. From Fig. 5, it is not surprising that each region's optimal emission-reduction rates under $\mathbf{U}_*^{\mathrm{NE}}$ are significantly lower than those under $\mathbf{U}^{\mathrm{w}}$. This is a direct reflection that under Nash equilibrium of the RICE game, the regions are working towards maximizing their own social welfare instead of a collective social welfare of all regions. As a result, each region has incentives to reduce its emission-reduction rate and its emission abatement cost, and thus improve its social welfare. These results partially show the rationale behind regions leaving signed climate treaties, e.g., Canada opt-out from Kyoto Protocol in 2012 (Nwanguma, 2016), because a less aggressive regional policy leads to higher economic benefit even facing climate change damages.

In Fig. 7, we plot the comparison of the atmospheric temperature deviation trajectories under $\mathbf{U}^{\mathrm{w}}$ and $\mathbf{U}_*^{(\mathrm{NE})}$. From the results, with non-cooperation, substantially higher atmospheric temperature deviation occurs under $\mathbf{U}_*^{(\mathrm{NE})}$, as a consequence of lower emission-reduction rates. For example, in 2620, the atmospheric temperature deviation resulted from RBA-DG is around 6°C while the atmospheric temperature deviation under global social welfare equilibrium is about 3°C.

6. CONCLUSIONS

In this paper, we investigated how cooperation and non-cooperation arise in regional climate policies under the RICE framework from the perspective of game theory and optimal control. The results reveal how game theory may be used to facilitate international negotiations towards consensus on regional climate-change mitigation policies, as well as how cooperative and competitive regional relations shape climate change for our future.

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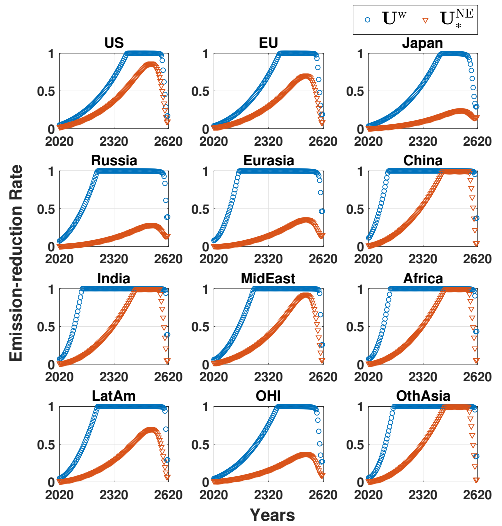

Fig. 5. Optimal emission-reduction rates under $\mathbf{U}^{\mathrm{w}}$ and $\mathbf{U}_*^{\mathrm{NE}}$.

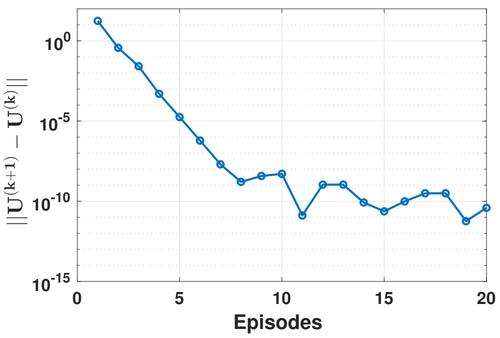

Fig. 6. Convergence of $\|\mathbf{U}^{(k+1)} - \mathbf{U}^{(k)}\|$ versus episodes.

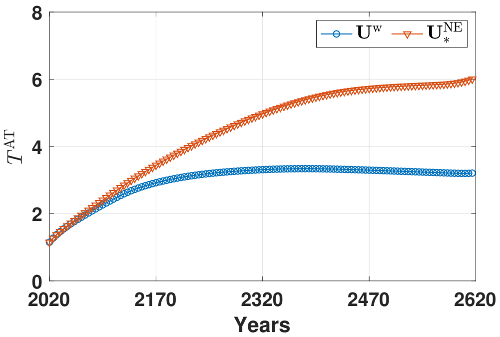

Fig. 7. Atmospheric temperature deviation trajectories under $\mathbf{U}^{\mathrm{w}}$ and $\mathbf{U}_*^{\mathrm{NE}}$.

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